

三维复杂区域对流换热问题的 控制方程及计算

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【摘要】 研究并发展了一套计算复杂区域对流换热问题的通用方法并开发了相应的可视化计算程序。针对所建立的一组以直角坐标速度分量表示的非正交曲线坐标系下的三维流动及传热控制方程, 采用有限体积方法离散, SIMPLER 算法求解, 并利用网格界面速度插值方法解决采用非交错网格时出现的压力波动问题。本文利用所开发的程序计算了低散热发动机冷却通道自然对流换热问题。

主题词: 曲线坐标 对流换热 数值模拟

分类号: O351 TK124

1 概 述

在求解不可压缩流动问题的方法中, 以 SIMPLE 算法^[1,2]为基础的一系列压力校正算法是行之有效的方法。作为 SIMPLE 算法的改进方法—SIMPLER 算法^[3], 由于压力直接由压力方程求出, 而不是由压力校正方法获得, 因此压力场能和速度场相协调, 有助于计算收敛。然而此方法的应用多针对正交坐标系。由于 SIMPLE 算法, 解决压力波动问题的一种常用方法是采用交错网格。但在其实施过程中存在以下问题: (1) 如果控制方程采用速度的直角坐标分量作为控制变量, 交错网格将失去其作用^[4]; (2) 由于各速度分量和其他标量存放在不同的网格节点上, 一方面造成程序编制的复杂性, 另一方面引入大量的插值, 从而影响计算精度及收敛性。迄今为止许多研究者采用非交错网格技术解决上述矛盾^[5,6]。文献[7]给出一种称之为网格界面速度插值的方法, 并通过一维和二维方程流动进行了验证。

本文借助一种守恒型变换方法建立了一组以直角坐标速度分量表示的非正交曲线坐标系下的三维不可压缩流动及传热方程, 并利用有限体积法及 5 种 3 点差分格式得到离散方程。本文将 SIMPLER 算法和网格界面速度插值方法的非交错网格技术推广到三维非正交曲线坐标系, 给出了相应的压力方程、压力修正方程、速度修正公式和逆变速度分量计算公式。

2 控制方程

在直角坐标系 y^i 中, 令 $y^0 = t$, 从而流体力学的基本方程组, 可以写为下列向量微分方程的守恒形式:

$$\partial_j / \partial y^j = f \quad (j = 0, 1, 2, 3) \quad (1)$$

其中

$$f = \begin{bmatrix} \rho \\ \rho f^1 \\ \rho f^2 \\ \rho f^3 \\ \rho f^E \end{bmatrix} \quad q_0 = \begin{bmatrix} \rho \\ \rho v^1 \\ \rho v^2 \\ \rho v^3 \\ \rho E^E \end{bmatrix} \quad q_1 = \begin{bmatrix} \rho v^1 \\ \rho v^1 v^1 \\ \rho v^1 v^2 \\ \rho v^1 v^3 \\ (\rho E + p) v^1 \end{bmatrix} - \frac{1}{Re} \begin{bmatrix} 0 \\ \tau^{11} \\ \tau^{12} \\ \tau^{13} \\ \tau^{1E} \end{bmatrix}$$

$$q_2 = \begin{bmatrix} \rho v^2 \\ \rho v^2 v^1 \\ \rho v^1 v^2 \\ \rho v^2 v^3 \\ (\rho E + p) v^2 \end{bmatrix} - \frac{1}{Re} \begin{bmatrix} 0 \\ \tau^{21} \\ \tau^{22} \\ \tau^{23} \\ \tau^{2E} \end{bmatrix} \quad q_3 = \begin{bmatrix} \rho v^3 \\ \rho v^3 v^1 \\ \rho v^3 v^2 \\ \rho v^3 v^3 \\ (\rho E + p) v^3 \end{bmatrix} - \frac{1}{Re} \begin{bmatrix} 0 \\ \tau^{31} \\ \tau^{32} \\ \tau^{33} \\ \tau^{3E} \end{bmatrix}$$

$$\tau^{iE} = v^1 \tau^{i1} + v^2 \tau^{i2} + v^3 \tau^{i3} + \frac{\gamma}{Pr} k \frac{\partial e}{\partial y^i}, \quad f^E = \rho f^j v^j, \quad E = \frac{1}{2} v^2 + e$$

其中: v^j 为直角坐标速度分量, τ^{ij} 为剪切应力张量, Pr 为 Prandtl 数, e 为单位质量的内能, k 为热传导率, γ 为比热比。

利用以 Jacobian 理论为基础的变换方法^[8], 可将(1)式变换成如下形式:

$$\partial Q_j / \partial \alpha^j = F \quad (j = 0, 1, 2, 3) \quad (2)$$

其中: $Q_j = (q_k / J) (\partial x^j / \partial y^k)$, $F = f / J$ ($j, k = 0, 1, 2, 3$)

(2) 式为非正交曲线坐标系下的流动控制方程。显然在 $f = 0$ 时, $F = 0$, 因此(1)式为强(或弱)守恒形式, (2)式也具有强(或弱)守恒形式, 变换没有改变方程的守恒特性。

为便于方程的离散, 将(2)式重新整理, 并用以下的统一形式表示:

$$\frac{\partial}{\partial \alpha} (\rho \Psi) + \frac{\Delta}{\Delta x^{(k)}} \left[\rho \Psi V^{(k)} - \Gamma_{\Psi} g^{(km)} \frac{\partial \Psi}{\partial m^{(m)}} \right] = S_{\Psi} \quad (3)$$

式中 $\frac{\partial}{\partial \alpha^{(j)}} = \frac{1}{g_{jj}} \frac{\partial}{\partial x^j}$, $\frac{\Delta}{\Delta x^j} = \frac{g_{jj}}{J} \frac{\partial}{\partial \alpha^{(j)}} \left(\frac{J}{g_{jj}} \right)$, $g^{(kj)} = \frac{g_{kk} g_{jj}}{g_{jj}}$

式中 Ψ 分别代表 v^j , T , K 和 ϵ , Γ_{Ψ} 为广义扩散系数, S_{Ψ} 为广义源项, g_{kj} 和 g^{kj} 分别为基本度量张量的协变分量和逆变分量。 $V^{(k)}$ 为曲线坐标逆变物理分量, 它和直角坐标分量 v_j 的关系为:

$$V^{(j)} = v^k \frac{\partial x^j}{\partial y^k} \frac{1}{g_{jj}} \quad (4)$$

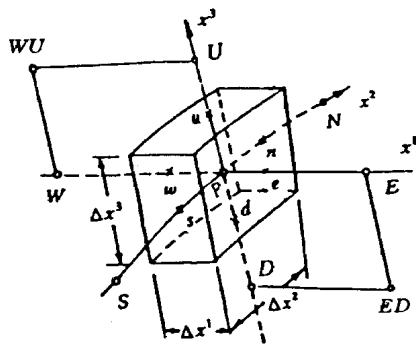


图1 计算域 P 点控制体

3 控制方程的离散及计算处理

3.1 控制方程的离散

对于(3)式, 本文采用有限体积法来导出离散方程, 即将(3)式在图1所示的微元控制体积积分, 并采用散度定理推得积分方程为:

$$\int \frac{(\rho \Psi)^n - (\rho \Psi)^0}{\delta t} J dx^1 dx^2 dx^3 + \int_w^n J \left[\rho V^1 \Psi - \Gamma_{\Psi} g^{1m} \frac{\partial \Psi}{\partial m^1} \right] dx^2 dx^3$$

$$+ \int_s^n J \left[\rho V^2 \Psi - \Gamma \Psi g^{2m} \frac{\partial \Psi}{\partial x^m} \right] dx^1 dx^3 + \int_d^u J \left[\rho V^3 \Psi - \Gamma \Psi g^{3m} \frac{\partial \Psi}{\partial x^m} \right] dx^1 dx^2 = \int S \Psi J dx^1 dx^2 dx^3 \quad (5)$$

将上式在非交错网格上, 并采用 5 种 3 点差分格式(中心差分、迎风格式、混合格式、指数格式和乘方格式)离散, 从而得到式(3)的最终离散方程^[8]:

$$A_P \Psi_P = A_E \Psi_E + A_W \Psi_W + A_N \Psi_N + A_S \Psi_S + A_U \Psi_U + A_D \Psi_D + S_C^\Psi \quad (6)$$

3.2 速度修正公式

设每一层次有一个压力的预估值或计算值 p^* , 代入动量方程中可以求得速度场 v^{j*} 。把改进后的压力值与 p^* 之差记为 p' , 与此相对应的速度修正值为 v^j , 则改进后的压力与速度分别为: $p = p^* + p'$ 和 $v^j = v^{j*} + v^j$ 。利用以上关系以及(6)式可得:

$$v_p^j = \sum_{nb} A_{nb}^j v_{nb}^j + B_{ij} \frac{\partial p'}{\partial x^i} \quad (7)$$

其中: $B_{ij} = -(\partial x^i / \partial y^j) \Delta x^1 \Delta x^2 \Delta x^3$ (不对 j 求和), $A_{nb}^j = A_{nb} / A_p^j$ 。略去(7)式中相邻节点速度修正值的影响后将该式代入 $v^j = v^{j*} + v^j$ 得:

$$v_p^j = v_p^{j*} + B_{ij} (\partial p' / \partial x^i) \quad (8)$$

利用速度的逆变分量 V^j 与直角分量 v^j 之间的关系, 并令: $C_{jk} = B_{ij} (\partial x^j / \partial y^k)$ 。从而得到:

$$V_p^j = V_p^{j*} + C_{jk} (\partial p' / \partial x^k) \quad (9)$$

(8)式和(9)式分别为速度直角分量和逆变分量的修正关系式。

3.3 压力修正方程和压力方程

压力修正方程可以通过连续性方程导出。将 $p = p^* + p'$ 代入(7)式所得的方程减去 p^* 代入(7)式所得的方程, 从而得到关于 p' 的代数方程:

$$A_P p' = A_E p'_E + A_W p'_W + A_N p'_N + A_S p'_S + A_U p'_U + A_D p'_D + b \quad (10)$$

$$b = [V^{1*} \Delta x^2 \Delta x^3]_w^e + [V^{2*} \Delta x^1 \Delta x^3]_s^n + [V^{3*} \Delta x^1 \Delta x^2]_d^u$$

由于本文采用的是 SIMPLER 算法, 因此压力方程也需通过连续性方程导出:

$$A_P p' = A_E p'_E + A_W p'_W + A_N p'_N + A_S p'_S + A_U p'_U + A_D p'_D + b_1 \quad (11)$$

$$b_1 = [\hat{V}^1 \Delta x^2 \Delta x^3]_w^e + [\hat{V}^2 \Delta x^1 \Delta x^3]_s^n + [\hat{V}^3 \Delta x^1 \Delta x^2]_d^u$$

其中: $\hat{V}^i$ 为假设逆变速度。

3.4 非交错网格技术

为消除采用非交错网格可能出现的不合理的压力波, 采用网格界面速度插值方法^[7]。以 w 界面为例说明该方法的应用。将(10)式和(11)式源项中的逆变速度分量用下式计算, 即:

$$V_w^1 = \bar{V}_w^1 + V_w^{1'} \quad (12)$$

其中 $\bar{V}_w^1$ 为主值, 它由下式计算: $\bar{V}_w^1 = \alpha V_w^1 + (1 - \alpha) V_p^1$, 式中 α 为线性插值系数, 其值在 0~1 之间。修正值 $V_w^{1'}$ 计算式为:

$$V_w^{1'} = V_w^1 - \bar{V}_w^1 = \beta (V_w^1 - \bar{V}_{w1}^1) + (1 - \beta) (V_w^1 - \bar{V}_{w2}^1) \quad (13)$$

其中 β 值为线性插值系数, 其值在 0~1 之间。 $V_w^1 - \bar{V}_{w1}^1$ 和 $V_w^1 - \bar{V}_{w2}^1$ 由下式计算:

$$V_w^1 - \bar{V}_{w1}^1 = (C_{11} / \delta x)_w (p_{mxP} - p_P), \quad V_w^1 - \bar{V}_{w2}^1 = (C_{11} / \delta x)_w (p_W - p_{mxP})$$

其中: $p_{mxP} = 0.5(p_W + p_E)$; $p_{mxW} = 0.5(p_{WW} + p_P)$

4 计算过程及算例

(1) 假定初始速度场 v^{j*} 和压力场; (2) 计算动量方程的系数, 并根据已知的速度计算假拟速度 $\hat{v}^i$; (3) 求解压力方程; (4) 把求解的压力作为 p^* , 求解动量方程, 得到 v^{j*} ; (5) 根据 v^{j*} 求解压力修正方程, 得到 p' ; (6) 利用压力修正值修正速度; (7) 利用修正后的速度计算温度 T ; (8) 返回到第二步重复计算, 直到收敛。

本文对以陶瓷为隔热材料的活塞式低散热发动机(俗称陶瓷发动机)散热场进行了计算, 其目的是希望深入了解该发动机的隔热效果, 并为进一步控制散热提供依据。图2为二分之一低散热发动机冷却通道计算区域网格, 计算网格数为 $48 \times 16 \times 20$ 。网格生成采用一种改进的直接控制网格节点分布的网格生成方法^[8]。边界热边界条件按如下方式给出: 对称面采用绝热条件, 固体壁面温度由实验给出。图3为瑞利数 $Ra = 10^7$ 时冷却通道内一些剖面的速度矢量图。图3中的速度是无量纲速度。在计算低散热发动机冷却通道自然对流换热问题时, 为便于计算, 将控制方程中的坐标、速度、压力、温度等无量纲化, 即瑞利数(Rayleigh) $Ra = g\beta(T_h - T_c)H^3/(\nu\alpha)$ 。式中 g 为重力加速度, β 为体膨胀系数, H 为冷却通道高度, T_h 和 T_c

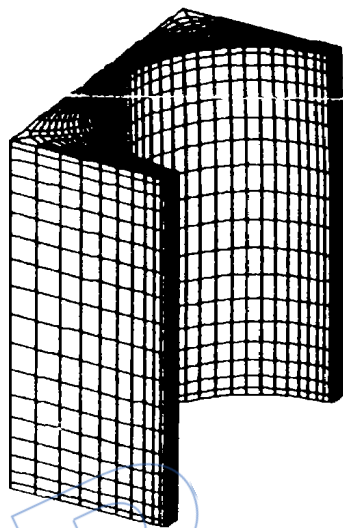


图2 二分之一冷却通道曲线网格

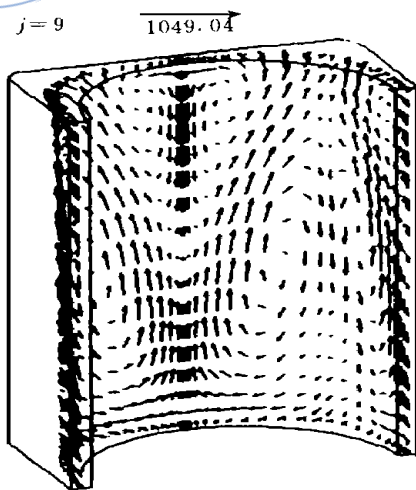
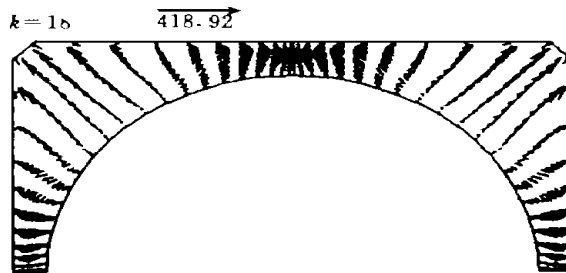
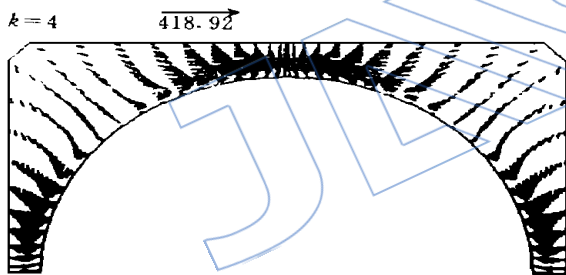


图3 $Ra = 10^7$ 时部分剖面上的速度矢量图

分别为冷却通道壁面最高温度和最低温度, ν 为运动粘性系数, α 为传热系数。可见, 通道内的流动是复杂的, 由于固体壁面温差的存在, 通道内形成了大的回流, 同时还伴随着小的回流。从温度和传热量的计算结果(图4)可见, 经由冷却通道空气散出的热量很少, 可以认为, 通道内空气起的是隔热作用。

由于算例是封闭腔室内稳态的自然对流问题, 所以没有进出口, 而且也没有物理上的初始条件。从软件运行上来讲, 将所有网格节点的最初温度设置为固体壁面温度的平均值, 而将这些节点的初始速度设置为零。

5 结 论

本文以 Jacobian 理论为基础的方程变换方法, 推导了非正交曲线坐标系下的流动及传热控制方程。通过和张量变换方法所得的方程比较可以看出, 本文采用的变换方法应用方便, 变换后的方程展开形式简单, 易于编程计算。本文对以直角坐标速度分量表示的曲线坐标系下的控制方程建立了一套计算方法, 包括有限体积方法结合 5 种 3 点格式对方程进行离散, 应用 SIMPLER 方法解决压力和速度耦合问题, 利用一种非交错网格技术解决压力波动问题。数值计算表明, 所建立的方法是行之有效的, 它为研究复杂区域对流换热问题奠定了基础。

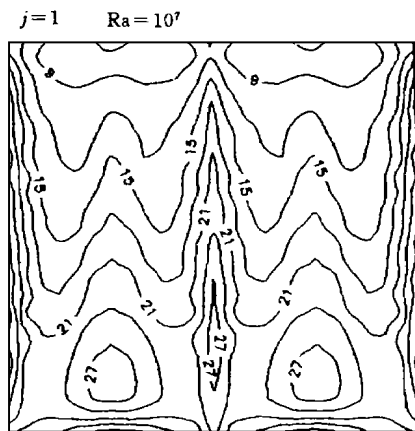


图 4 展开截面局部 Nussell 数分布

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CONTROL OF BLADE TIP CLEARANCE WITH SHAPE MEMORY ALLOYS

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ABSTRACT

An adaptive sealing ring is developed to control the blade tip clearance of compressors with the help of shape memory alloys. Such a ring can change the blade tip clearance in adaption to the working conditions of engine. Its control mechanism is described. The structural design of the adaptive sealing ring is completed. The deformation of the adaptive ring is calculated and analyzed according to the constitutive relation of shape memory alloys.

Key Words Shape memory alloys Tip clearance Sealing rings

NUCLEATE BOILING OF BINARY MIXTURE AT HIGH HEAT FLUX UNDER MICROGRAVITY

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ABSTRACT

An analytical model for nucleate pool boiling heat transfer of binary mixtures in high-flux regime is formulated according to Helmholtz instability theory of vapor-liquid interface. The nucleate pool boiling for binary mixtures at high heat flux under microgravity is also characterized by the existence of a liquid layer known as macrolayer between the heater surface and the vapor mass. The dual-bubble layer structure can maintain up to CHF condition due to less buoyancy. The expressions for fraction of bubble contact area on the heater surface and initial macrolayer thickness are derived in term of the thermophysical properties or/and the heat flux. A CHF model based on hydrodynamics is provided. The predicted results are compared with experimental data and theoretical analysis. Some important conclusions are drawn as follows: The fraction of bubble contact area on the heater surface is independent of the input heat flux, but much greater than that at terrestrial conditions; Initial macrolayer thickness is much greater than that at terrestrial conditions due to less velocities of vapor phase and liquid phase; The CHF values for binary mixtures under microgravity are about 60~80% of those at terrestrial conditions, but are much greater than those of single components under microgravity.

Key Words Heat transfer Boiling Bubble dynamics of boiling

CALCULATION OF HEAT CONVECTION IN COMPLEX 3D GEOMETRIES

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ABSTRACT

A simulation method is presented for heat convection in 3D complex geometries. For a non-orthogonal coordinate formulation of the governing equations with cartesian velocity components, pressure and temperature, a finite volume method is uti-

lized for discretization, SIMPLER algorithm is applied to pressure– velocity coupling, and the cell– face velocity interpolation method is employed for overcoming the pressure oscillations associated with the nonstaggered grid system. A visualization computing software is developed and applied to prediction of the heat convection in the cooling passage of low heat rejection engine.

Key Words Curvilinear coordinates Convective heat transfer Numerical simulation

DROPWISE CONDENSATION AS A TYPICAL FRACTAL GROWTH PROCESS

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ABSTRACT The dropwise condensation is a typical fractal growth process, according to the experimental data and conclusions published in last 30 years. The drop size distribution in dropwise condensation is deduced, based on the random fractal model. The fractal dimension index in the size distribution is determined with renormalization method. The analytical results are in good agreement with experimental data.

Key Words Dropwise condensation Drop Characteristic dimensions
Random distribution

A STUDY ON AVERAGED HEAT TRANSFER CHARACTERISTICS OF A ROTATING CAVITY WITH PRESWIRLING INFLOW

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ABSTRACT A turbine cooling configuration of an aeroengine was simplified to a rotating cavity with a tangential preswirling fringe guide vane and a radial outlet. The influences of rotating Reynold number, cooling air Reynold number and Grashoff number on the averaged heat transfer characteristics were experimentally investigated. The experiment indicated that the flow pattern in the rotating cavity is mainly controlled by Rossby number. The averaged heat transfer coefficient on the main disk increases with the increase of cooling air Reynold number and Grashoff number, the increase of rotating Reynold number decreases the heat transfer coefficient at some regions of the Rossby number and causes it to increase at others. Two experimental correlations for different regions of Rossby number, are established for the averaged heat transfer coefficient on the main surface.

Key Words Swirling Discs Heat transfer